

# Numerical Models of the Volume Response Function of Conductance Catheters: the Effect of Multiple Unused Electrodes

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**Abstract:** A large animal finite element model for a multi-electrode conductance catheter is presented for use in the right ventricle. The concept of conductance measurement for the determination of blood pool volume is explained, and the nonlinearity of the volume response function is determined for a commercially available conductance catheter. The effect of unused electrodes made in a tetrapolar measurement using the catheter is shown.

**Keywords:** Finite element, conductance measurement, tetrapolar electrode, ventricular volume and quasi-static.

## 1. Introduction

Conductance technology has been utilized as an invasive tool to detect instantaneous left and right ventricular blood volume since 1982. Conductance measurement is usually performed using a 4 electrode (tetrapolar) catheter placed in the heart chamber of interest to make a measurement. The operating principle is that a device generates an electric field using a current source, and blood pool volume is determined by calculating the conductance from the instantaneous differential voltage signal. A quasi-static model is presented for the estimation of volume (RVUs) by modeling the catheter electrodes submerged in a cuvette of saline.

The earliest formula for calculating volume from a time varying conductance signal is,

$$V(t) = \frac{\rho L^2}{\alpha} \cdot (G(t) - G_{||}) \quad (1)$$

where  $V$  is the time varying volume signal,  $\rho$  is the blood resistivity,  $L$  is the distance between the voltage electrodes used on the catheter,  $\alpha$  is a unitless scaling factor,  $G(t)$  is the time varying conductance signal measured, and  $G_{||}$  is the parallel conductance, usually from the surrounding tissue contributing to the signal [1]. In equation (1), if  $\alpha$  is constant, it implies a linear relationship between the conductance signal and the volume derived from it. It has been shown previously that the value of  $\alpha$  is not a constant; but is in fact nonlinearly dependent on the conductance  $G(t)$  [2]. It is usually assumed that this nonlinear dependence is strongest in larger animals like swine and dogs, but may not be necessary for smaller animals like mice [3].

Modeling of the *in vivo* measurement is rarely done because of the complex nature of the surrounding structures, and the uncertainty of the continually changing properties within the living organism. However, modeling of the *in vitro* case is still valuable because it can provide confirmation that the catheter is performing as expected before parallel conductances are introduced.

Calibration in various sizes of cuvettes is established as a laboratory practice for determining volume in relative volume units (also known as RVUs). It is this calibration practice which is modeled here. The relationship between absolute volume as measured between the voltage sensing electrodes of a catheter, and the conductance measured can be used to ensure that the estimated volume is correct by determining the true value of  $\alpha$ .

## 2. Methods

### A. Geometry and meshing

The type of catheter which is modeled is a standard commercial 6 French 12 segment catheter (Millar Instruments Model SPC-562-1, Houston, TX) for use in animals with a 7 mm electrode spacing (with the exception of the first 3 electrodes, which are 3 mm apart), as shown in Fig. 1. This catheter geometry is usually used to perform volume studies on pigs or dogs. Only 6 electrodes are shown in the model, because 6 electrodes are all that are needed to span the right ventricle, which is one of the most common places to catheterize in pacemaker research. The catheter is most sensitive to changes in field geometry between the sensing electrodes, so the more proximal electrodes are omitted. If the most distal electrode is number 1, and the electrodes are numbered consecutively, electrodes 1 and 6 are current stimulation electrodes, while electrodes 2 and 5 are for voltage sensing. Electrodes 3 and 4 are not used in this measurement.

Even though only 4 electrodes are used in the measurement (2 for stimulation via a current source, and 2 for voltage measurement), the other electrodes are not omitted. The walls of the cuvette are moveable to simulate a standard cuvette calibration at 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, and 200 mL.

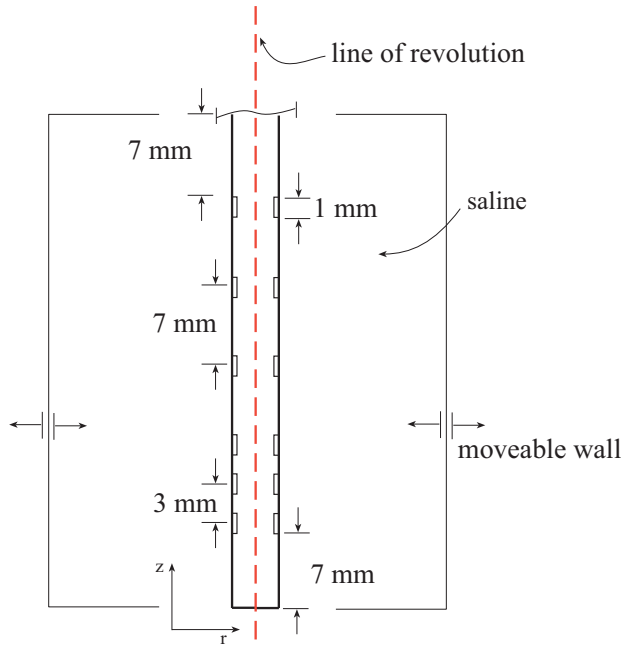


Fig. 1. FEM model 2D axisymmetric geometry; distal section of catheter.

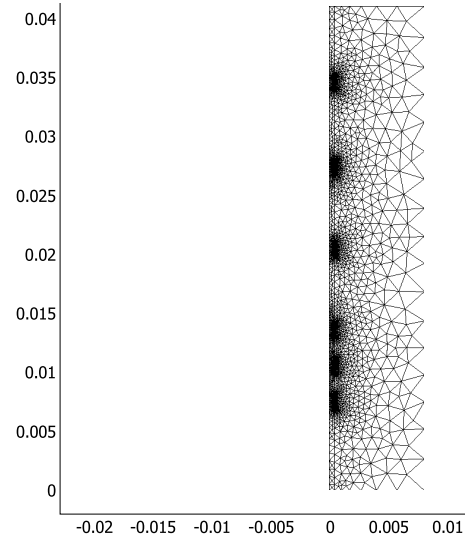


Fig. 2. 2D axisymmetric mesh.

The mesh shown in Fig. 2 was refined near the electrodes automatically by the meshing software because of the smaller geometry present. It is also necessary to further refine the mesh near the electrodes because this area near the electrode will have the most non-uniform current density. The mesh refinement is shown in detail in Fig. 3 below. The number and size of elements in the model varies based on the geometry supplied by the size of the saline container, but the minimum element quality is found in the largest cuvette, and furthest from the catheter. All mesh parameters are summarized in Table 1.

Table 1: Mesh Properties.

number of mesh points	5109
number of elements	10028
number of boundary elements	756
minimum element quality	0.5793
element area ratio	3.46E-05

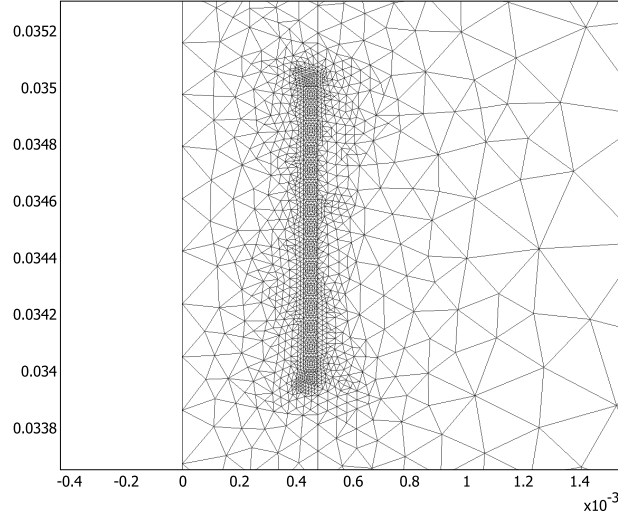


Fig. 3. Close up view of refined mesh at one electrode.

### B. Finite Element Model (FEM) Definition

The quasi-static meridional electric currents mode in Comsol is used to set up and solve the solution space. The governing equation for this model is,

$$-\nabla \cdot [(\sigma + j\omega\epsilon_r\epsilon_0)\nabla V] = 0, \quad (2)$$

where  $\sigma$  is the conductivity,  $\omega$  is the angular frequency,  $\epsilon_r$  is the relative permittivity,  $\epsilon_0$  is the permittivity of free space, and  $V$  is the electric scalar potential. Equation (2) is a form of Poisson's equation and can be derived from Gauss's law and the continuity equation as described previously [4].

The electrical properties used in the model are in Table 2 below. In a cuvette, the walls of the container are normally made of plastic, which is approximated in the model as the electric insulating boundary. The catheter body is made of plastic and is placed in saline of conductivity  $\sigma = 1$  S/m, which is close to the conductivity of blood.

Table 2: Electrical Properties.

domain	$\sigma$ (S/m)	$\epsilon$ (F/m)
saline	1	7.08E-10
catheter body	0	1.95E-11
electrode	9.09E+06	8.85E-12

### C. Boundary Conditions

The governing equation assumes the model will be submerged in an insulated space, so the boundary condition for the cuvette wall is set as an electric insulation boundary,

$$n \cdot [(\sigma + j\omega\epsilon_r, \epsilon_0)\nabla V] = 0. \quad (3)$$

In the AC/DC quasi-static mode in Comsol, there are only two ways to model the electrode stimulation on the boundary. The first, is as a current source by defining the current density, and then second is as a voltage source. Even though the stimulating electrodes are actually current sources, it is not sufficient to set the current density to a constant value. This is because by setting the current density to a single value, we are assuming that the current density will be uniform across the entire surface (which is not true). Instead, the model uses an ODE setting to force the voltage to be equal to a value that minimizes the equation

$$\int \vec{J}_n \cdot d\vec{S} - I_0 = 0 \quad (4)$$

where  $I_0$  is the desired stimulating current, while  $J_n$  is the current density flowing out of the source electrode. The integral is taken in Comsol as an integration coupling variable active only in the domain of the electrode. The result of equation (4) is to stimulate using a current of the user's choosing ( $I_0$ ), while not forcing the current density to be uniform. On the stimulating electrodes, the boundary condition is set as a voltage source, where  $V$  is forced to be a source at a frequency of 19218 Hz, close to the 20 kHz used in many commercial conductance measurement instruments.

## 3. Results

### A. Field Uniformity and its effect on volume measurement

The high conductivity of the electrodes that are unused in the system forces the current density to increase near the surface of the metal. This is an important point which is often overlooked in catheter measurement. Were the electrodes not present, the field would be completely uniform near the center of the catheter in a relatively small cuvette like the one in Fig. 4 (a).

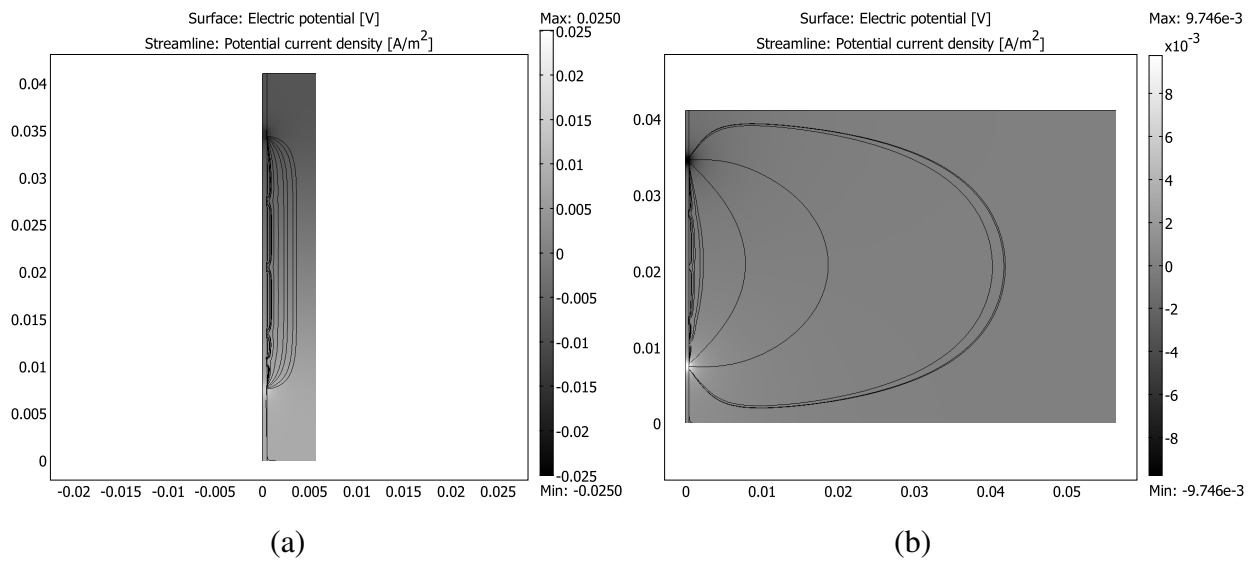


Fig. 4. (a) 1 mL cuvette and (b) 100 mL cuvette current densities and potentials developed.

As seen in Fig. 4 (a), as the volume of the cuvette measured between the voltage electrodes decreases, the linearity of the field improves, allowing a closer estimate of true volume than in Fig. 4 (b), where the current density spreads out to better fit the shape of the container. This phenomenon is the reason for the nonlinear relationship between conductance and volume. The fact that there is a nonlinear relationship makes sense because as the volume of a container increases, the conductance signal measured will eventually reach a saturation point. The point where the conductance will no longer increase even though the volume can increase is known as G infinity, or the conductance of a volume of infinite size. This concept is illustrated with data from the model in Fig. (5).

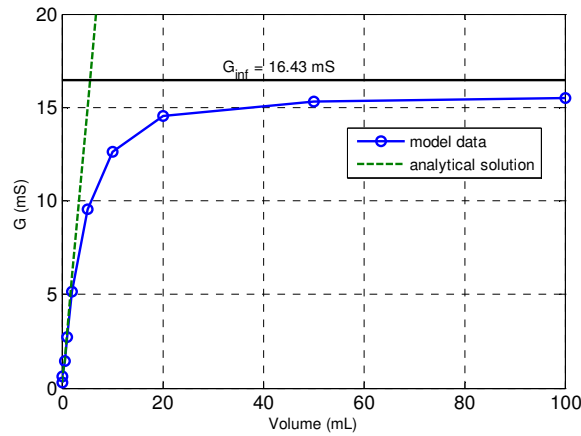


Fig. 5. Nonlinearity of the conductance - volume relationship.

The analytical solution is calculated with a constant  $\alpha$ , scaled to match the lower volume data. Had the unused electrodes not been present, the lower volume data would be exactly linear. The relationship between the conductance and the volume is nonlinear, which directly implies that  $\alpha$  is nonlinear. This nonlinearity can be seen again in Fig. 6, where the model results are fit to a quadratic curve. The point where  $\alpha = 0$  is analogous to the concept of G infinity, and it is obvious from the linear fit that the value would be overestimated were  $\alpha$  assumed to be linear. Some work has been done which shows the value of alpha to approach 1 at low conductances [2]. This work was done on a mouse catheter, where there were no intermediate unused electrodes. This fact, in addition to the difference in spacing between electrodes, is enough to reduce the value of  $\alpha$  to 0.3 as G approaches 0.

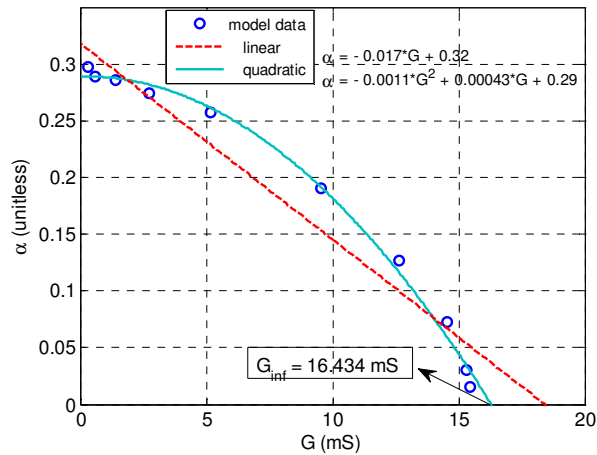


Fig. 6. Alpha conductance relationship.

It is interesting to note here that many researchers still use a constant value of  $\alpha$  in larger animal models because they are unaware that there is a nonlinear dependence to  $\alpha$ . The effect is only less pronounced in small volumes compared to the length  $L$  from equation (1) because the smaller the volume relative to the size of the catheter spacing, the closer a linear relationship can approximate the true nature of  $\alpha$ . The approximate value of the volume in the right ventricle of swine is anywhere from 30-50 mL [5]. This model implies that with the current catheter geometry, right ventricular volume far exceeds the approximately linear region for  $\alpha$ .

#### 4. Conclusion

The nonlinearity of  $\alpha$  is confirmed for larger animal models by these results. We can conclude that it can be inaccurate to use a constant or even linearly varying value for  $\alpha$  for any size animal, but especially so in dogs, pigs, and humans. The existence of extra unused electrodes was shown to substantially alter the current density near the catheter, and could therefore be a significant factor when interpreting results. The value of  $\alpha$  will never equal 1 because of the effect of the unused electrodes on the field. This is of particular importance because so many assume that at small volumes it has a value of exactly 1 (which is true only for models with no extra electrodes). The model can be used to determine if the volume being measured is within the linear region for a given assumption of constant  $\alpha$ , and also to determine if  $G$  infinity is sufficiently larger than what would be measured by a given catheter design.

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